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Numerical Solution Of The SIRV Model Using The Fourth-Order Runge-Kutta Method

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Abstract**:** *This study aimed to predict the spread of the Covid-19 virus in Maluku Province using the fourth-order Runge-Kutta method. The mathematical model of the spread of the Covid-19 virus is a system of differential equations which includes Susceptible (S) variables, namely human subpopulations that are susceptible to Covid-19 virus infection, Infected (I), namely human subpopulations infected with the Covid-19 virus, Recovered (R) namely sub-population of people who have recovered and Vaccination (V) namely a subpopulation that has been vaccinated and is immune to the Covid-19 virus, used as initial values. The values of* $k, \alpha, \beta, \gamma, \psi, \mu, \mu_c$ *are parameter values that are numerically solved by the fourth-order Runge-Kutta method performed for 24 literations with* $h = 0{,}01$. *Data were obtained from the Maluku Provincial Health Office from March 2022 - November 2022. Based on the data obtained, the average of the data is used as the initial value, where* $S = 198.890, I = 204, R = 172$, and $V = 7.693$. *The initial and parameter values were substituted into the numerical solution and simulated using Matlab. The rate value of each class for the next 24 months for the Susceptible (S), Infected (I), and Recovered (R) classes has decreased until it approaches zero equilibrium. It shows that the subpopulation of the three classes no longer exists, and the Vaccinated (V) class has increased significantly because almost all of the population has been vaccinated in the next 24 months. It shows that after an individual is vaccinated, he does not return to being vulnerable.*

Keywords*: Epidemic Model, Covid-19, Vaccination, Fourth Order Runge-Kutta Method, Numerical Solution.*

1. Introduction

Coronaviruses encompass a family of viruses known to induce a spectrum of illnesses, ranging from common colds to more severe conditions such as acute respiratory syndrome (SARS). The identification of Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) as the causative agent for a cluster of pneumonia cases in Wuhan, China, was reported by Chen et al. (2020) in late 2019. Subsequently, the virus disseminated not only within China but also globally. By February 2020, the World Health Organization (WHO) officially declared coronavirus disease 2019 (COVID-19) as a worldwide pandemic. (*Situation Report-51 SITUATION IN NUMBERS Total and New Cases in Last 24 Hours*, n.d.)

There has been no cure found for Covid-19, so vaccines are one of the new initiatives launched to strengthen one's immunity. The emergence of vaccines has given many people hope to be free from the plague soon. Vaccination aims to activate antibodies so that they are immune to disease or only suffer from the mild disease (Liu et al., 2022). The mathematical model was first constructed by Kermack and Mckendrick (Kermack & Mckendrick, 1927).

Therefore, more and more researchers are considering mathematical models in solving this Covid-19 spread case. Ndairou et al. also developed a mathematical modeling system to study the spread of COVID-19 in Wuhan, China (Ndaïrou et al., 2020). Prem et al. investigated the status of COVID-19 in Wuhan (Prem et al., 2020). In another analysis, Hellewell et al. identified successful isolation techniques for COVID-19 (Hellewell et al., 2020). Kucharski et al. proposed a mathematical model-based analysis of COVID-19, where the authors considered all positive cases in Wuhan, China, until March 5, 2020 (Kucharski et al., 2020). Liu et al. Examined the possible numerical assessment of the early reproductive rate of COVID-19 in their research. (Liu et al., 2020) Fanelli and Piazza utilized mathematical modeling to explore and forecast the onset of COVID-19 in the three most impacted countries in March 2020 (Fanelli & Piazza, 2020). In a recent study, Chakraborty and Ghosh (2020) explored a hybrid ARIMA-WBF model for predicting the spread of COVID-19 in various countries worldwide. Shereen et al. (2020) delved into the characteristics, regional impact, and distribution of coronavirus within the human population. On a related note, Tahir et al. (2019) focused on investigating Middle East Respiratory Syndrome (MERS) caused by a coronavirus in humans. Additionally, Zhao and Chen (2020), as mentioned by Zhao et al., contributed to the study.Utilized mathematical modeling to examine the scope of the coronavirus pandemic. Shim et al. extended their study by investigating the transmission dynamics of 2019-nCoV in South Korea (Shim et al., 2020). Nuraini, N. et al., in their study, predicted that the end of endemic transmission of Covid-19 in Indonesia would end in April 2020, and the total number of cases would exceed 8000 (Nuraini et al., 2020). In the research, Nesteruk formulated an SIR epidemic model, statistically analyzed the parameters employed in the suggested model, and demonstrated strategies for managing these infections. (Nesteruk, 2020). Zu et al. used the sensitivity analysis method (Zu et al., 2020). They conducted predictions on the epidemic trend and transmission risk associated with COVID-19, assessing the effectiveness of various intervention strategies. Tang et al. (2020) specifically delved into the efficacy of quarantine and isolation measures, thereby determining the trajectory of the COVID-19 epidemic in the later stages within China. Ahmed et al. analyzed the mathematical model of COVID-19 in their study using numerical approaches and logistic models. They proposed three numerical techniques to solve the equation: the Euler method, second-order (RK2), and fourth-order (RK4) Runge-Kutta method (Ahmed et al., 2020). In another study by Okuonghae and Omame, they used numerical simulation to investigate the

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effect of control measures, especially social distancing, mask use, and case detection (contact tracing and subsequent testing) on COVID-19 (Okuonghae & Omame, 2020). Meanwhile, Foy et al. used the SEIR model to evaluate age-specific vaccine distribution strategies in India (Foy et al., 2021). On the other hand, research on an effective vaccine against COVID-19 continues at full speed. Several academic studies have been conducted in this context. Kaur and Gupta pointed out in their study that different types of vaccine strategies have been developed against COVID-19 and provided an overview of research to find an effective vaccine against the novel coronavirus affecting the global world, economy, health, and human life (Kaur & Gupta, 2020)

In this study, a new model is introduced that refers to the modified model (Rijoly et al., 2022) in the form of adding a new variable of vaccine subpopulation class that shows the individuals who have been vaccinated. The benefit of vaccines for the human body is to stimulate the immunity of the human body and to protect them from Covid-19 viruses. Concentrating on the model, this study endeavors to identify the equilibrium points for each compartment in both disease-free and endemic states. Additionally, the research explores the stability of these equilibrium points obtained. (Sapulette, Lesnussa & Rijoly, 2023). The last study was to use the numerical simulation for this model.

2. Methods

This type of research is quantitative. The data used in this study were sourced from the Maluku Provincial Health Office and the Maluku Provincial Statistics Agency. The data required includes the number of subpopulations in Maluku Province, the number of subpopulations born, the number of human subpopulations susceptible to Covid-19 infection, the number of human subpopulations who have been infected with Covid-19, the number of human subpopulations who have recovered or who have become immune to Covid-19, the number of human populations who died from Covid-19 infection, The count of subpopulations that have received vaccinations and developed immunity to Covid-19, along with the life expectancy of individuals in Maluku Province in the year 2022.
The research steps are as follows:

1. Collecting data
2. Finding the parameters of the model that has been made
3. Inputting parameter values in the model based on the data obtained
4. Solving the SIRV model with fourth-order Runge-Kutta.
5. Numerical simulation using Matlab
6. Drawing conclusions on the numerical solution of the SIRV model using fourth-order Runge-Kutta.

3. Result and Discussion

The SIRV model described in this research consists of 4 subpopulations, namely subpopulation *S* (Susceptible human subpopulation), subpopulation *I (*Human subpopulation that has been infected with Covid-19), Population *R (*Human subpopulation that has recovered from Covid-19), and subpopulation *V* (Vaccinated human subpopulation) with the following assumptions (Sapulette, Lesnussa & Rijoly, 2023):

a. The natural mortality rate was present in each subpopulation and was assumed equal to μ.
b. The susceptible subpopulation (*S*) might increase due to births, which was assumed to be k.
c. μ_c is a death rate due to the interaction of individuals in the susceptible subpopulation (*S)* with individuals in the infected subpopulation (*I*) cause by Covid-19.
d. Individuals in the *Infected* (*I*) subpopulation moved to the *Recovered* (*R*) subpopulation with a recovery rate of β because they received treatment.
e. Those in the Recovered (*R*) category would shift to the Vaccination (*V*) subgroup as part of the process of receiving vaccination to reinforce their immunity.
f. Individuals within the susceptible subpopulation (*S*) may also choose to get vaccinated to avoid contracting Covid-19, with a vaccination rate equivalent to ψ.
g. Those who had recovered from Covid-19 and received vaccination would no longer be susceptible to the virus.

Using these assumptions, the illustration of the spread of Covid-19 is diagrammatically depicted in the compartmental section, as indicated in Figure 1.

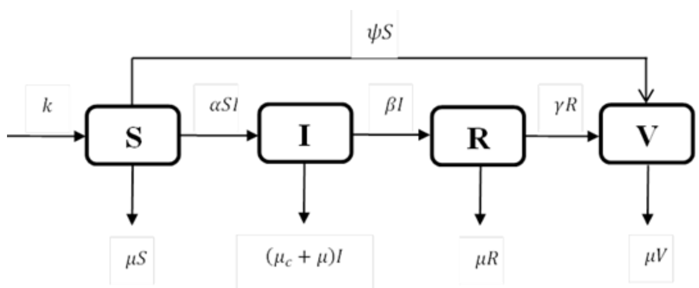

Figure 1. SIRV model

We construct a mathematical model of Covid-19 with respect to the above diagram. We write it as the following equations.

$$\frac{dS}{dt} = k - \mu S - \alpha SI - \psi S \tag{1}$$
$$\frac{dI}{dt} = \alpha SI - (\mu_c + \mu)I - \beta I \tag{2}$$
$$\frac{dR}{dt} = \beta I - \mu R - \gamma R \tag{3}$$
$$\frac{dV}{dt} = \gamma R - \mu V + \psi S \tag{4}$$

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3.1 Parameters

Numerical simulation of the SIRV model used the Fourth Order Runge-Kutta Method in predicting the spread of Covid-19 with vaccination in Maluku Province. The parameter values used in this study are as follows.

1. Individual birth rate
$$k(t) = \frac{Number\ of\ babies\ born}{Total\ resident\ population}$$

2. Rate of susceptible individuals becoming infected individuals
$$\alpha(t) = \frac{The\ average\ number\ of\ infected\ patients}{number\ of\ the\ population}$$

3. The recovery rate of infected individuals
$$\beta(t) = \frac{The\ average\ number\ of\ cured\ patients}{The\ average\ number\ of\ infected\ patients}$$

4. Vaccination rate of individuals before the infection
$$\psi(t) = \frac{Average\ number\ of\ patients\ vaccinated\ before\ infection}{number\ of\ population}$$

5. Vaccination rate of individuals after infection
$$\gamma(t) = \frac{Average\ number\ of\ patients\ vaccinated\ after\ infection}{average\ number\ of\ infected\ patients}$$

6. Natural mortality rate
$$\mu = \frac{1}{Life\ Expectancy}$$

7. The death rate due to Covid-19 infection
$$\mu_c = \frac{The\ average\ number\ of\ patients\ who\ died\ from\ infection}{average\ number\ of\ infected\ patients}$$

The natural mortality rate of individuals can be computed by considering the life expectancy of the population in Maluku Province. Data obtained from the Central Bureau of Statistics of Maluku Province for the year 2022 indicates that the life expectancy of individuals in this region was 66.09 years. (Statistics Agency of Maluku Province, 2023).

Based on the calculation of the parameter values above, the parameter values used in determining the numerical solution in the SIRV model for the spread of the Covid-19 virus with vaccination are obtained which are summarized in Table 1. below:

Table 1. Parameter Estimation

Parameters	Definition	Value
k	Rate of individual births	0,01676
α	The rate at which individuals who are susceptible become infected.	0,00011
β	The rate at which individuals who are infected recovery	0,84314
γ	The rate at which individuals receive vaccination following infection	0,84314
ψ	The rate at which individuals are vaccinated prior to infection	0,00404
μ	The rate of natural death among individuals.	0,00126
μ_c	The rate of death attributed to Covid-19 infection	0,01471

3.2 Simulating Numerical Model with Fourth Order of Runge-Kutta Method

Simulation of the SIRV model used the Fourth Order Runge-Kutta method. This simulation was done by substituting the initial value in the form of secondary data and parameter values that had been determined. The data used in the simulation is presented in the following table 2:

Table 2. Covid-19 Patient Data and Vaccination Data of Maluku Province for the Period March 2022 - November 2022

Date	**Variables**			
	S	**I**	**R**	**V**
March 1 - 31, 2022	1.834.651	1.195	1.134	25.646
April 1 - 30, 2022	1.850.026	75	65	12.460
May 1 - 31, 2022	1.858.310	43	24	4.249
June 1 - 30, 2022	1.855.491	25	22	7.088
July 1 - 31, 2022	1.858.351	44	14	4.217
August 1 - 31, 2022	1.858.644	112	65	3.805
September 1 - 30, 2022	1.854.910	97	73	7.546
October 1 - 31, 2022	1.862.145	56	42	383
November 1 - 30, 2022	1.858.489	186	111	3.840
TOTAL	**1.790.009**	**1.833**	**1.550**	**69.234**
Average	**198.890**	**204**	**172**	**7.693**

Based on data obtained at the Health Office of Maluku Province from March to November 2022, it is known that the average subpopulation *S (Susceptible)* is 198,890 peo-

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ple, subpopulation *I (Infected*) are 204 people, subpopulation *R (Recovered)* are 172 people, and subpopulation *V (Vaccination)* is 7,693 people. Therefore, the initial value is obtained as in table 3.

Table 3. Initial Values

Variables	**Value**	**Description**
S_i	198.890	Data on the number of human subpopulations susceptible to Covid-19 in the Maluku Provincial Health Office
I_i	204	Data on the number of human subpopulations that have been infected with Covid-19 at the Maluku Provincial Health Office
R_i	172	Data on the number of human subpopulations that have recovered after Covid-19 at the Maluku Provincial Health Office
V_i	7.693	Data on the number of human subpopulations that have received vaccinations at the Maluku Provincial Health Office

The results of iteration of numerical solutions using the Fourth Order Runge-Kutta method up to t = 24 months using Matlab. The iteration results for the rate of susceptible individuals *S*, *I*, *R*, and *V* are shown in the following graph.

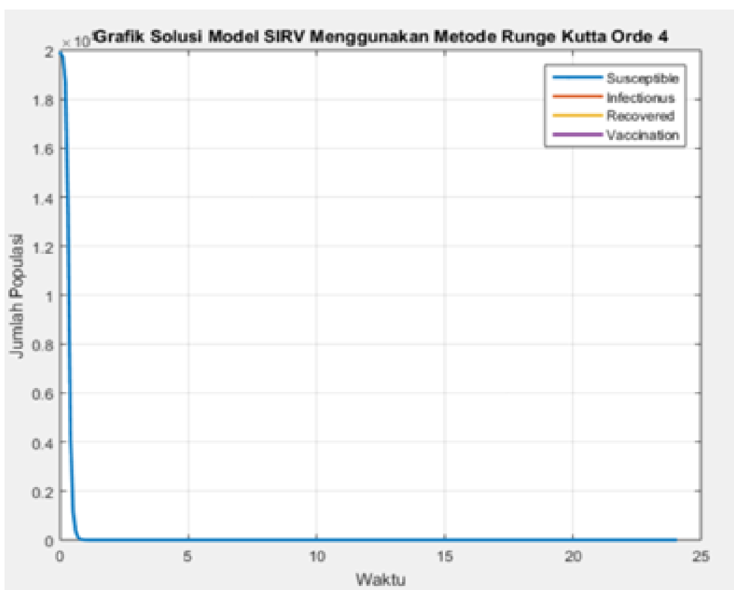

Figure 2. Subpopulation Rate Graph of Vulnerable Individuals *(S)*

From Figure 2. We can see that the predicted rate of the vulnerable individual class *(S) has* decreased. In the 24th month, the number of vulnerable individuals is no longer there because many individuals have been vaccinated to strengthen immunity so as not to become vulnerable.

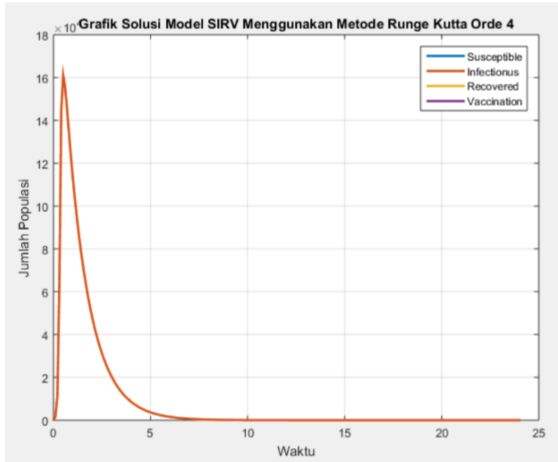

Figure 3. Subpopulation Rate Graph of Infected Individuals *(I)*

Interpreting Figure 3, we can see that the predicted rate of the infected individual class *(I)* decreased before the 5th month. Because vulnerable individuals are reduced and absent, infected individuals are also absent.

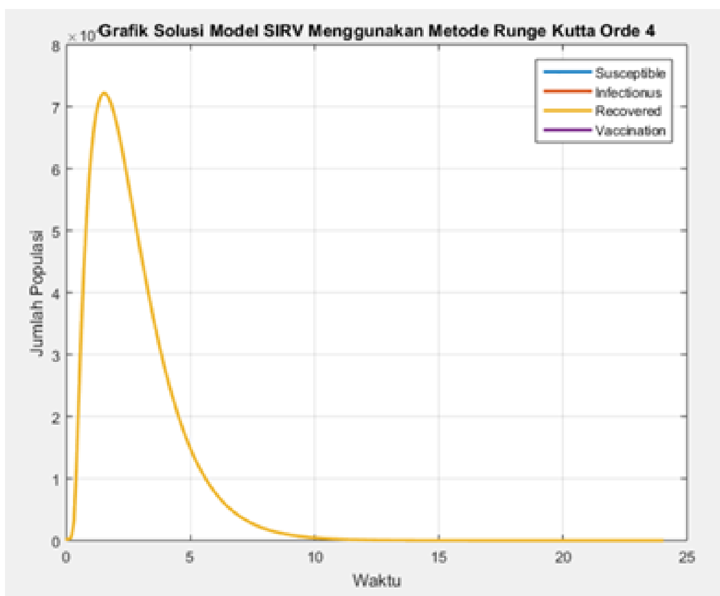

Figure 4. Subpopulation Rate Graph of Recovered Individuals *(R)*

Based on Figure 4, we can see that the prediction of the class rate of recovered individuals *(R)* has decreased before the 5th month because the number of infected individuals has been declared absent, so those who recover from Covid-19 also do not exist.

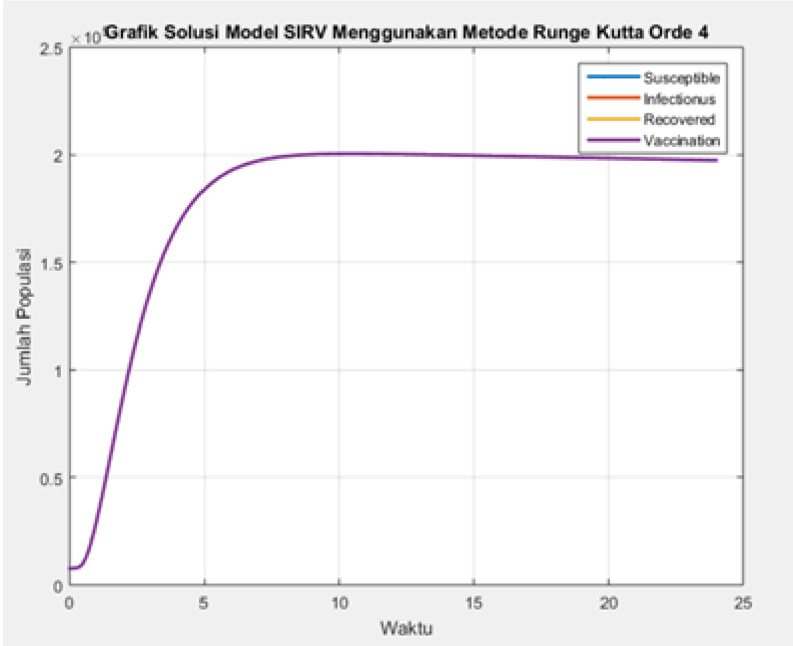

Figure 5. Subpopulation Rate Graph of Vaccinated Individuals *(V)*

Based on Figure 5, we can see that that the predicted rate of vaccinated classes *(V)* increases before month five because the number of vaccinated individuals is increasing in Maluku Province.

4. Conclusion

The conclusions of this study are as follows:

The Fourth Order Runge-Kutta method was used to numerically solve the SIRV model, simulating the spread of the Covid-19 virus. This SIRV model comprises Susceptible, Infected, Recovered, and Vaccinated populations. From the model, the Matlab Software iteration calculation was carried out with the initial value of $S_0 = 198.890, I_0 = 204, R_0 = 172,$ dan $V_0 = 7.693$. Then, iterations were carried out starting from $t_0 = 0$ month to $t_{24} = 24$ months.

Prediction using Matlab Software obtained results where, in the next 24 months, vulnerable individuals (*S*), infected individuals (*I*), and recovered individuals (*R*) no longer exist, and vaccinated individuals amounted to 197,490 people. It means that the subpopulations (*S*), (*I*), and *(R)* will decrease slowly for the next 24 months while the population (*V*) will increase slowly for the next 24 months. So, it can be inferred that the spread of Covid-19 in Maluku Province is influenced by the vaccination rate.

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